

## Abstract

This paper provides a new context for an established metaphysical debate regarding the problem of persistence. I contend that perdurance (the claim that objects persist by having temporal parts) can be precisely formulated in quantum mechanics due to an analogy with spatial parts, which I claim correspond to the decomposition of the quantum state provided by a localization scheme. However, I present a ‘no-go’ result that rules out the existence of an analogous temporal localization scheme, and so argue that quantum objects cannot be said to perdure. I conclude by surveying the remaining metaphysical options.

# Do Quantum Objects Have Temporal Parts?

1st March, 2012.

**1. Introduction** This paper provides a new context for an established metaphysical debate regarding the *problem of persistence*. Namely, how can it be said that one and the same physical object persists through time while changing over time? I contend that a popular view about persistence which maintains that objects persist by *perduring* – that is, by having temporal parts – receives a particularly neat formulation in quantum mechanics due to the existence of a formal analogy between time and space. However, on closer inspection this analogy fails due to a ‘no-go’ result which demonstrates that quantum systems can’t be said to have temporal parts in the same way that they have spatial parts. Therefore, if quantum mechanics describes persisting physical objects, then those objects cannot be said to perdure.

This argument serves two aims. The first is to continue the recent tradition of addressing the problem of persistence in the context of specific physical theories: Balashov (2010) considers special relativity; Butterfield (2005, 2006) considers classical mechanics. The second aim is to provide a novel interpretation of the no-go result mentioned above, which is well-known in the quantum foundations literature but rarely discussed by philosophers of physics. The result is often phrased like this: There exists no time observable canonically conjugate to the Hamiltonian. This fact was first observed by Pauli in 1933, and there are

various proofs which arrive at this conclusion.<sup>1</sup> I claim this result is best understood not as an argument against the existence of time (Halvorson 2010) but rather as an argument that quantum systems do not have (proper) temporal parts.<sup>2</sup>

*1.1. Argument Outline* The argument takes the form of a *modus tollens* which I give a sketch of here, leaving technical details for later sections. I begin with a characterization of *perdurantism* as the thesis that objects persist through time just as they stretch through space.

**perdure** The part-whole relation for persisting objects applied to time works just like the part-whole relation with respect to space. That is, persisting objects have proper temporal parts associated with an arbitrary division of the times over which the object persists.

I then argue that ordinary quantum mechanics describes persisting objects. This claim requires (at least) a robust scientific realism about quantum mechanics.

**quantum** A quantum object (an isolated system described by a ray in Hilbert space undergoing unitary evolution) is a persisting object.

Taken together, **perdure** and **quantum** imply that quantum objects have temporal parts. Call this view *quantum perdurantism*. I further claim that if quantum mechanics does describe

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<sup>1</sup>It was recently observed that Pauli's proof admits a significant class of counterexamples (Galapon 2002). The result I will give is instead related to the proofs of Srinivas and Vijayalakshmi (1981); Halvorson (2010).

<sup>2</sup>This is not to say that there are not other valid interpretations. For example, Unruh and Wald (1989) provide an argument against the existence of an ideal quantum clock.

persisting objects, then it also provides a legitimate account of the spatial parts of such an object.

**spatial** The part-whole relation in space for quantum objects is given by the *spectral decomposition* of the Hilbert space into orthogonal subspaces, provided by the position operator  $\hat{Q}$ . This decomposition is unique, and is known as a localization scheme. A quantum object has spatial parts *iff* there exists a localization scheme.

Since **perdure** asserts that the relation of parthood applied to time is just like the relation of spatial parthood, it follows that a quantum perdurantist is committed the existence of a *temporal* localization scheme which operates in an analogous way to **spatial**. That is, **perdure**, **quantum** and **spatial** jointly entail the following conditional statement.

**temporal** If quantum perdurantism is true then every persisting quantum object has a unique decomposition into temporal parts provided by a temporal localization scheme.

Unfortunately for the perdurantist, the consequent is demonstrably false: The *spectral condition* states that the Hamiltonian of every system has a spectrum bounded from below – roughly, every system has a state of lowest energy – and entails that no quantum objects possess a temporal localization scheme. Therefore quantum perdurantism is false; quantum objects have no (proper) temporal parts.

This leaves two possibilities: Either they have no temporal parts (*endurantism*), or one temporal part (*temporal holism*). I argue that, although there is little to choose between these rival views in the context of non-relativistic quantum mechanics, considerations from relativity favor temporal holism.

**2. The Metaphysics of Persisting Objects** How does a material object persist *through* time while changing *with* time? There are essentially two schools of thought: Either a persisting object has no temporal parts, is self-identical at every moment it exists, and its spatial properties change with time (*endurantism*), or a persisting object necessarily has temporal parts which have differing spatial properties (*perdurantism*). Another common way of phrasing the distinction is as a conflict between *three-dimensionalism* and *four-dimensionalism*: If an object endures then it exists in three dimensions (since it has no temporal width); if it perdures then, having temporal width, it exists in four dimensions.<sup>3</sup>

I will follow Lewis (1986) in using the term perdurance for the latter possibility, which I take to be a thesis about the existence of temporal parts.

Something perdures iff it persists by having different temporal parts, or stages, though no one part of it is wholly present at more than one time; whereas it endures iff it persists by being wholly present at more than one time. Perdurance corresponds to the way a road persists through space; part of it is here and part of it is there, and no part is wholly present at two different places. (Lewis 1986, 202)

So for perdurantism to be true, it must be the case that persisting objects be amenable to decomposition into (proper) temporal parts. It has been complained that the notion of *being wholly present* is problematic (e.g. (Sider 1997; McCall and Lowe 2003)) but I will argue that, within quantum mechanics, it can be given a precise meaning due to a formal analogy with *being wholly located*.

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<sup>3</sup>Among four-dimensionalists, there is a further dispute about reference: When we speak of an object e.g. “the table” do we refer to a particular instantaneous *stage*, i.e. *the table at time t*, (Sider 1997; Hawley 2004) or the entire temporally extended object?

I take the motivation for perdurantism to be a strong analogy between time and space. Sider expresses this idea as follows:

As I see it, the heart of four-dimensionalism [perdurantism] is the claim that the part-whole relation behaves with respect to time analogously to how it behaves with respect to space ... Applied to time, the idea is that for any way of dividing up the lifetime of an object into separate intervals of time, there is a corresponding way of dividing the object into temporal parts that are confined to those intervals of time. (Sider 1997, 204)

I will argue that the appropriate part-whole relation for spatial parts in quantum mechanics is provided by a localization scheme (in Section 4), which commits the perdurantist to a thesis about temporal *localizability* in quantum mechanics (in Section 5).

**3. Persisting Objects in Quantum Mechanics** Quantum mechanics provides our best theory of matter, and its empirical predictions have been startlingly accurate. That much is uncontroversial. On the other hand, any attempt to assert exactly *why* it has proved so successful, or precisely what it tells us about the nature of material objects involves taking sides on disputes regarding its interpretation that have lasted over 80 years and show little sign of abating. Therefore, I will proceed by specifying under what conditions one would be committed to regarding the quantum state as describing a persisting material object. Nonetheless, I take it that *prima facie* a realist metaphysician who takes chairs (composed of complex collections of organic molecules) to be persisting objects would be compelled to similarly regard, say, a molecule of Buckminsterfullerene ( $C_{60}$ ) composed of sixty atoms of carbon, and recently shown to display distinctly quantum behavior (Nairz et al. 2003).

First, some details about the formalism of ordinary (non-relativistic) quantum mechanics. As our concern is with spatio-temporal properties, we will consider systems with no internal degrees of freedom (*i.e.* spinless particles). Therefore, the *state space* of the theory is provided by the space of square integrable functions defined over all of space, that is, infinite-dimensional (separable) complex Hilbert space  $\mathcal{H} = L^2(\mathbb{R})$  (for simplicity we will consider only one spatial dimension). The *pure* states  $|\psi\rangle$  are in one-to-one correspondence with the one-dimensional subspaces of  $\mathcal{H}$  or, equivalently, the set of normalized vectors that individually span those subspaces. Since  $\mathcal{H}$  is a vector space, linear combinations of pure states are also pure states (this is known as the *superposition principle*). In what follows I will only consider pure states.

The first interpretative posit I require is *realism*, the claim that real physical systems are authentically described by quantum mechanical states. The next posit I require is *completeness*, the claim that a pure state provides a complete description of an individual quantum system which leaves nothing out (*i.e.* no hidden variables). So far we would be justified in claiming that the quantum state describes a physical object. But what about *persisting* objects?

We require some facts about quantum dynamics. In the Schrödinger picture, the history of a system is given by a series of (pure) states  $|\psi(t)\rangle$ , where  $t \in \mathbb{R}$ . Once the state  $|\psi(0)\rangle$  at a time  $t = 0$  is given, the entire history is determined according to the time-dependent Schrödinger equation in terms of a one-parameter (strongly continuous) group of unitary operators  $U(t) = e^{-iHt}$ , where  $H$  is the Hamiltonian of the system. If a pure state  $|\psi(0)\rangle$  describes a physical object which exists at time  $t = 0$ , then a history  $|\psi(t)\rangle$  describes a persisting object which exists at each  $t$  and changes with time. The infamous measurement problem arises when we consider the relation of the unitary dynamics of the state to the

results of laboratory observations.

The *observables* of the system are self-adjoint operators<sup>4</sup> on  $\mathcal{H}$  associated with measurable quantities, and the values they may take on measurement correspond to the *spectrum* of the operator. For an observable  $\hat{A}$  with a discrete spectrum (*e.g.* the Hamiltonian of a simple harmonic oscillator), each spectral value  $a_n$  has an associated *eigenvalue* equation  $\hat{A}|\phi_n\rangle = a_n|\phi_n\rangle$ , where  $|\phi_n\rangle$  is an *eigenstate* of  $\hat{A}$ . Distinct eigenvalues are associated with mutually orthogonal subspaces (*eigenspaces*)  $|a_n\rangle$  which are spanned by the vectors  $|\phi_n\rangle$  with zero inner product,  $\langle\phi_m|\phi_n\rangle = 0$  for  $m \neq n$ . Any vector can be written as a weighted sum  $|\psi\rangle = \sum_n |a_n\rangle\langle a_n|\psi\rangle = \sum_n c_n|\psi_n\rangle$ , where  $|\psi_n\rangle$  is the *projection* of  $|\psi\rangle$  onto  $|a_n\rangle$  and  $c_n$  are complex coefficients  $\sum_n |c_n|^2 = 1$ . This is known as the *spectral decomposition* or *resolution of the identity* of  $\mathcal{H}$  with respect to  $\hat{A}$ , which we can write as  $\hat{A} = \sum_n a_n |a_n\rangle\langle a_n|$ .

According to the standard story, the probability of obtaining a particular value  $a_n$  in measurement is given by  $\langle\psi|\psi_n\rangle = |c_n|^2$  (the *Born Rule*) and, having observed a system to take a particular value, upon repeating the measurement of the observable it will be found to have the same value  $a_n$ . However, according to the formalism the only way this could happen is if the system were in an eigenstate of  $\hat{A}$  (known as the *eigenstate-eigenvalue link*), but since (i) in general a system is not in an eigenstate with respect to  $\hat{A}$ , and (ii) the dynamics provided by the Schrödinger equation are unitary, there is (in general) no reason to think that a system should ever be found in such a state. This is the measurement problem.

The third posit I will require is, therefore, that we consider *isolated* quantum systems which need only unitary evolution for their complete description over time; persisting quantum objects are isolated systems on this view. This means that we will not need to

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<sup>4</sup>An operator  $\hat{A}$  is symmetric on  $\mathcal{H}$  iff  $\langle\psi|\hat{A}\phi\rangle = \langle\hat{A}\psi|\phi\rangle$  for all elements in its domain  $\mathcal{D}(\hat{A}) \subseteq \mathcal{H}$ . It is self-adjoint  $\hat{A} = \hat{A}^\dagger$  iff it is symmetric and  $\mathcal{D}(\hat{A}) = \mathcal{D}(\hat{A}^\dagger)$ .



concern ourselves with the measurement problem. This invites the worry that very few systems in the actual world will fall under this criterion. Maybe so, but on at least one interpretation of quantum mechanics (Everett-style realist ‘no-collapse’) *all* systems undergo only unitary evolution.

**4. Parts and Spatial Parts** What is a part of a quantum object? I contend that a suitable part-whole relation is provided by considering the subspaces of  $\mathcal{H}$ , or equivalently the projections onto those subspaces. According to classical mereology, the relation of *parthood* is (minimally) reflexive (everything is part of itself), transitive (if  $p$  is part of  $q$  and  $q$  is part of  $r$  then  $p$  is part of  $r$ ) and antisymmetric (no two distinct things can be part of each other). As is well known, the subspaces of a vector space  $A, B, C, \dots$  are partially ordered by the relation of *inclusion*, which is reflexive ( $A \subseteq A$ ), transitive (if  $A \subseteq B$  and  $B \subseteq C$  then  $A \subseteq C$ ) and antisymmetric (if  $A \subseteq B$  and  $B \subseteq A$  then  $A = B$ ).

I claim that in quantum mechanics the *spatial* parts are given in terms of the subspaces of the  $\mathcal{H}$  associated with the spectral decomposition of the position observable  $\hat{Q}$ . As  $\hat{Q}$  has a (purely) continuous spectrum, this will require some more details about self-adjoint operators. Since the pioneering work of von Neumann we have known that any self-adjoint operator (even an unbounded continuous operator) on  $\mathcal{H}$  is uniquely associated (up to unitarity) with a *spectral measure* which allows us to replace the sum over projections onto eigenspaces with an integral  $\hat{Q} = \int_{\mathbb{R}} \lambda dE_{\lambda}$ , where  $E_{\lambda}$  is a spectral family of projections with  $\lambda \in \mathbb{R}$ . It is this which allows us to write the position operator as an integral over space  $\hat{Q} = \int dx x |x\rangle\langle x|$ .<sup>5</sup>

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<sup>5</sup>However, note that the equation  $\hat{Q}|q\rangle = q|q\rangle$  is a merely formal expression in this case *i.e.*  $|q\rangle$  is a (so-called) *improper* eigenstate and not an element of  $\mathcal{H}$ .

We may associate with each Borel set<sup>6</sup>  $\Delta \in \mathfrak{B}(\mathbb{R})$  a projection  $P^{\hat{Q}}(\Delta) = \int_{\Delta} \lambda dE_{\lambda}$ . The map  $P^{\hat{Q}} : \Delta \mapsto P^{\hat{Q}}(\Delta)$  is known as a Projection Valued Measure (PVM) and has the properties (i)  $P^{\hat{Q}}(\mathbb{R}) = 1$  (normalization), and (ii)  $P^{\hat{Q}}(\bigcup_n \Delta_n) = \sum_n P^{\hat{Q}}(\Delta_n)$  (strong  $\sigma$ -additivity), where  $\Delta_n$  is a sequence of mutually disjoint Borel sets  $\Delta_m \cap \Delta_n = \emptyset$  for  $m \neq n$ . We can do this quite generally since the self-adjoint operators on  $\mathcal{H}$  are in one-to-one correspondence with the set of PVM's (Teschl 2009, Thm. 3.7).

What does this have to do with spatial parts? Well, the Borel sets correspond to spatial regions in a very intuitive way since any two sets of spatial points which occupy the same volume of space are assigned the same Borel set (of  $\mathbb{R}^3$  now); a Borel set is an *equivalence class* of sets of points under the relation *having the same volume*. Take an ordinary object that occupies exactly a cube. Pick four opposite vertexes of the cube which lie in a plane (such that not all four are on the same face). How many ways are there of dividing the cube into two parts of equal volume along that plane? Presumably we would want to say: "There is only one way, straight down the middle!" And this is just the answer we find from looking at the Borel sets.

However, if we consider instead the set of points that lie in the interior of the cube there are *three* ways: one that excludes the points that lie on the plane, one that gives them to the left hand part, and one that gives them to the right hand part.<sup>7</sup> The upshot of these sort of

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<sup>6</sup> $\mathfrak{B}(\mathbb{R})$  is the smallest  $\sigma$ -algebra over  $\mathbb{R}$  containing all open intervals of  $\mathbb{R}$ . A  $\sigma$ -algebra over a set is a (nonempty) collection of subsets closed under complementation and countable union.

<sup>7</sup>Also note that to allow spatial parts corresponding to non-measurable sets would open the door to paradoxical results regarding their re-composition, illustrated by the Banach-Tarski theorem. For a discussion of these issues see Arntzenius (2008).

considerations, I take it, is that we would rather associate spatial parts with Borel sets rather than sets of points (or, equivalently, only with sets of points dense in some open interval of  $\mathbb{R}$ ).

So, if you accept the notion of parthood I articulated above, then the PVM  $P^{\hat{Q}}$  associated with the position observable  $\hat{Q}$  provides a neat assignment of spatial regions to parts of the state space. It has the attractive property that any spatial region is associated with a unique projection, and, furthermore, if two regions are disjoint  $\Delta_1 \cap \Delta_2 = \emptyset$  then they are associated with mutually orthogonal projections  $P^{\hat{Q}}(\Delta_1)P^{\hat{Q}}(\Delta_2) = P^{\hat{Q}}(\Delta_2)P^{\hat{Q}}(\Delta_1) = 0$ , while if they overlap  $\Delta_1 \cap \Delta_2 \neq \emptyset$  then their intersection has the unique projection  $P^{\hat{Q}}(\Delta_1)P^{\hat{Q}}(\Delta_2) = P^{\hat{Q}}(\Delta_2)P^{\hat{Q}}(\Delta_1) = P^{\hat{Q}}(\Delta_1 \cap \Delta_2)$  (from property (ii)).

This is a *localization scheme* in the sense that performing a measurement that corresponds to a projection  $P^{\hat{Q}}(\Delta)$  has the possible outcomes  $\{0, 1\}$ : either the system is located in  $\Delta$  or the system is not located in  $\Delta$ . Furthermore, these possibilities are mutually exclusive in that  $P^{\hat{Q}}(\mathbb{R}/\Delta) = I - P^{\hat{Q}}(\Delta)$  (from (ii)). Therefore, the system may be said to be ‘wholly located in  $\Delta$  at  $t$ ’ on the condition that  $P^{\hat{Q}}(\Delta)|\psi(t)\rangle = |\psi(t)\rangle$ . Since in general the system will not be in an eigenstate of *any* projection  $P^{\hat{Q}}(\Delta)$  we say not that it is located somewhere but rather that it is *localizable*. If a pure state  $|\psi\rangle$  describes a quantum object then, I claim, the projections  $P^{\hat{Q}}(\Delta)|\psi\rangle$  denote the spatial parts of the object.

Another characteristic of the PVM  $P^{\hat{Q}}$  which justifies the contention that it provides an assignment of parts is that it *covaries* with spatial translations  $U(a)^{-1}P^{\hat{Q}}(\Delta)U(a) = P^{\hat{Q}}(\Delta + a)$ , where  $U(a) = e^{-i\hat{P}a}$  is the one-parameter unitary group of spatial translations generated by the total momentum  $\hat{P}$ . Roughly, this is a consequence of the fact that  $\hat{Q}$  and  $\hat{P}$  are canonically conjugate  $[\hat{Q}, \hat{P}] = i\hbar$ . Viewing these transformations passively as a relabeling of the spatial axis, *covariance* assures us that we are picking out the same parts despite having changed their relation to the labels.

Now, there is something a little disconcerting about these spatial parts. Firstly, (in general) each quantum object appears to be composed of parts that together cover all of space (from (i)). Secondly, these spatial parts do not ‘move with the object’ since  $U(t)^{-1}P^{\hat{Q}}(\Delta)U(t) = P^{\hat{Q}}(\Delta)$ . Neither of these features represent genuine problems for this view. First, there is nothing metaphysically necessary about the view that physical objects have limited spatial extent. Fields, for example, qualify as genuine physical entities without being limited to a particular region of space. Moreover, although the localization scheme necessarily covers all of space, the object itself may be localized in the above sense. Second, this might be thought of as a boon for the perdurantist since it restores a symmetry between time and space by removing the need to define spatial parts relative to spatial *location* (see Butterfield (1985)).

Another potentially disconcerting feature of these quantum spatial parts is that they are defined in terms of the position observable for the *entire* system with state  $|\psi\rangle$ , and so it may be the case that even though we (naively) suppose the system to be further decomposed into distinct subsystems  $|\psi\rangle = |\eta\rangle \otimes |\xi\rangle$ , the spatial parts assigned in this way fail to respect this decomposition such that the spatial degrees of freedom of the subsystems fail to be independent. This is known as *entanglement*, and is a pervasive feature of quantum mechanics. If the subsystems are considered to be spatially separated then entanglement may result in non-locality, in the sense that the results of local measurements of position on one subsystem may depend on the results of local (but distant) measurements on the other subsystem. The view taken here is that this apparent tension results from an incorrect notion of mereology: subsystems do not correspond to independent spatial parts *unless* they are associated with mutually orthogonal projections  $P^{\hat{Q}}(\Delta)$ .

**5. (No) Temporal Parts** What is a *temporal* part of a quantum object? A possible response might go as follows: Since the instantaneous quantum state determines all the kinematical properties of the system, we can specify the temporal parts of a quantum object by a simple assignment of the states  $|\psi(t)\rangle$  to times  $t \in \mathbb{R}$ . This ‘naive’ scheme would assign to arbitrary sets of times  $\{T\}$  the temporal part  $|\psi(t)\rangle$  only if  $t \in \{T\}$ . However, the scheme completely fails to provide a partition of the object into *parts*. The problem is that the parameter  $t$  indexes a family of temporal *translations*, so fails to respect the requirement that temporal parts be ‘wholly present’ at  $t$ .

By means of analogy, consider the family of states  $|\psi(a)\rangle = U(a)|\psi(0)\rangle$  where  $U(a)$  is again the group of spatial translations by  $a$ . The naive spatial location scheme would assign parts (subspaces of  $\mathcal{H}$ ) to spatial points  $\{X\}$  according to whether or not value of the index  $a$  lies in  $\{X\}$ . But the claim that  $|\psi(a)\rangle$  is ‘wholly located’ at the position  $a$  doesn’t make sense since  $a$  merely denotes the spatial interval by which the state  $|\psi(0)\rangle$  was translated. In general  $|\psi(0)\rangle$  will not be located anywhere in particular (unless an eigenstate of some  $P^{\hat{Q}}(\Delta)$ ) and so  $|\psi(a)\rangle$  picks out the same ‘part’ as  $|\psi(0)\rangle$ .

Exactly the same analysis applies to the temporal translations  $U(t)$ . So to identify distinct temporal *parts* the perdurantist needs a temporal localization scheme which assigns to temporal intervals (proper) subspaces of the state space of the system, and so parts of  $|\psi\rangle$ : that is, a Projection Valued Measure  $P^{\hat{T}}(\Delta)$  associated with a self-adjoint operator  $\hat{T}$ . In order that this scheme picks out genuine temporal parts, we should expect this scheme to *covary* with time translations  $U(t)^{-1}P^{\hat{T}}(\Delta)U(t) = P^{\hat{T}}(\Delta + t)$  so that under a relabeling of the time axis the labels change but not the parts.

Unfortunately for the would-be quantum perdurantist, it turns out that these requirements are in conflict with the restriction on physical Hamiltonians known as the *spectral condition*,

which permits only Hamiltonian operators with a spectrum bounded from below *i.e.* only systems with a state of lowest energy. The usual argument for this is that to do otherwise would allow for systems which may transfer energy to their surroundings indefinitely. While it is true that all systems we know obey the spectral condition (*e.g.* a free particle or harmonic oscillator), we could also view it as a principle of the theory on par with the second law of thermodynamics.

Now, it is a theorem that if a self-adjoint Hamiltonian on  $\mathcal{H}$  obeys the spectral condition then there can be no time PVM that covaries with time translations (see Srinivas and Vijayalakshmi (1981, Thm. 1) or Halvorson (2010)). Roughly, the spectral condition implies that any two vectors in  $\mathcal{H}$  related by a time translation are non-orthogonal, so that the only assignment of temporal intervals to mutually orthogonal subspaces is  $P^{\hat{T}}(\Delta) = 0$  for all  $\Delta$ .<sup>8</sup> Thus no quantum object has (proper) temporal parts.

It is worth emphasizing that the problem is not that we cannot find a covariant assignment of temporal intervals to operators, but rather that there is no such assignment to projections on  $\mathcal{H}$ . So while it *is* the case that we can find a covariant mapping of intervals to operators in the form a Positive Operator Valued Measure (POVM), these assignments come without an associated spectral decomposition of  $\mathcal{H}$  and, moreover, are non-unique (Hegerfeldt and Muga 2010). This failure to find a temporal decomposition of the state space into distinct subspaces means that a quantum object cannot have temporal parts in the same way as it has spatial parts.

However, such POVM's do provide a *generalized* resolution of the identity parameterized by  $t$  so giving meaning to the notion of a temporal interval in terms of operators on  $\mathcal{H}$  (Holevo 1982). Therefore I see no reason to deny that time should be afforded the status of a physical parameter – although not one associated uniquely with a self-adjoint operator – and in this

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<sup>8</sup>This implies there is no self-adjoint operator  $\hat{T}$  canonically conjugate to the Hamiltonian.

sense I disagree with Halvorson's claim that "time [in quantum theory] is not a quantity at all – not even an unobservable quantity" (Halvorson 2010, 1).

**6. Conclusion** For the perdurantist, the failure to find a temporal localization scheme has a worrying implication for the claim that persisting quantum objects have temporal parts: If an object has temporal parts then times should be associated uniquely with subspaces of  $\mathcal{H}$  just as spatial regions are uniquely associated with spatial parts through a localization scheme  $P^{\hat{Q}}(\Delta)$ . The claim that the part-whole relation applied to space is the same as the part-whole relation applied to time is demonstrably false when applied to quantum objects. To the extent that we have reason to think that all persisting objects are quantum objects, this provides reason to doubt that perdurantism is true.<sup>9</sup>

In fact, the result we have demonstrated that only two temporal partitions are consistent with the requirements: Either there are no temporal parts, or there is one part corresponding to the entire history  $|\psi(t)\rangle$ . While I have argued that both these options are problematic for the perdurantist, the former is consistent with endurantism since the endurantist maintains that persisting objects have no temporal parts and no temporal width; the endurantist claims there is exactly one persisting object existing at each moment, and so may consistently attribute to that object at time  $t$  the state  $|\psi(t)\rangle$ .

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<sup>9</sup>Arguably, classical persisting objects are best thought of as "patterns that emerge from an ubiquitous, continuous, and very efficient process of decoherence." Butterfield (2006, 41). Decoherence refers to the process by which interactions between an 'object' system (*e.g.* a dust particle) and its environment serve to pick out a dynamically 'preferred' basis according to which the object system is approximately diagonalized. My argument concerns the basis independent description of the entire system of object *and* environment.

However, the latter option admits a valid four-dimensional interpretation which I call *temporal holism*, corresponding to the idea that the quantum state has exactly one temporal part comprising its entire history.<sup>10</sup> This offers an interesting resolution of the problem of persistence since it effectively denies that persisting objects change with time. It is distinct from endurantism in that although the same object is present at each time it is never wholly present; and distinct from perdurantism in the sense that although the persisting object exists at many times, no part of it is ever wholly present either.

A similar view has been advocated by Rovelli (2004) on the basis of relativistic considerations that he traces back to Dirac's preference for the Heisenberg formulation of quantum mechanics (in which the observables not the state are regarded as varying in time) over the Schrödinger picture (adopted above, in which the state varies not the observables). Since the Schrödinger and Heisenberg pictures are strictly equivalent within non-relativistic quantum mechanics the situation there is effectively neutral with respect to temporal holism and endurantism. Nonetheless, temporal holism may be thought to win out to the extent that four-dimensionalism is encouraged by relativity, having ruled out perdurantism by the above argument.

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<sup>10</sup>This is similar to the 'worm view' advocated by Balashov (2010) in the context of special relativity but there are obvious difficulties with describing quantum systems in terms of world-tubes.



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